

Ch. 12 Lines and Planes

Big Problem-Solving Idea:

Two questions guide my problem solving:

- What do you WANT?
- What are you GIVEN?

The goal is to understand and visualize, not memorize dozens of problem types.

Chapter 12 will be broken into three parts

Want?

Part 1: Lines need a POINT and DIRECTION.

Part 2: Planes need a POINT and NORMAL.

Given?

Part 3: How to use what is given.

Ch. 12 Lines and Planes

Part 1: Lines in 3D

Let (x, y, z) be any point on the line.

Let $\mathbf{r} = \langle x, y, z \rangle$ be the “position” vector that points from the origin to this point.

To describe a line, we want...

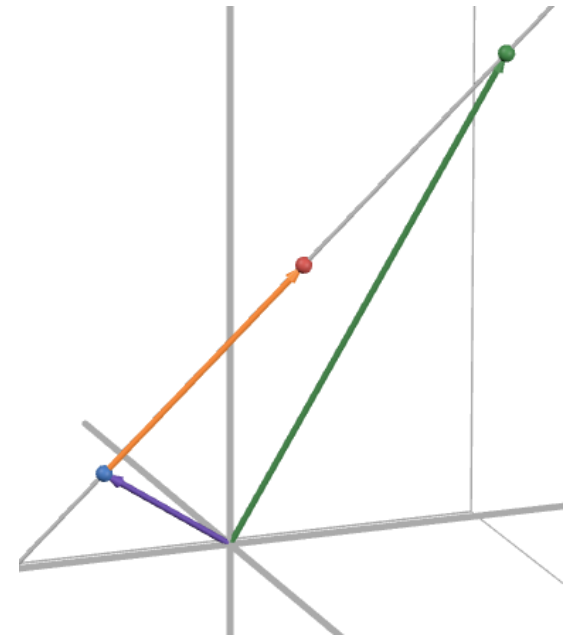
A direction vector (parallel to the line) and an initial position (pt on the line).

$$\mathbf{v} = \langle a, b, c \rangle$$

$$\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$$

then for some scalar, t , we can write

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$



Line equations forms

- Vector:

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$

- Parametric:

$$(x, y, z) = (x_0 + at, y_0 + bt, z_0 + ct)$$

- Symmetric:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Basic Example — Given Two Points

Find parametric equations of the line through P(3,0,2) and Q(-1,2,7).

Parallel, Intersecting, and Skew Part 1 HW #5–7

General Line Facts

1. Parallel if their directions are parallel.
2. Intersect if they have an (x,y,z) point in common.
3. They are **skew** if they do not intersect and are not parallel.

Examples of Each...

12.5 part 2 Planes in 3D Part 2 HW #1–3

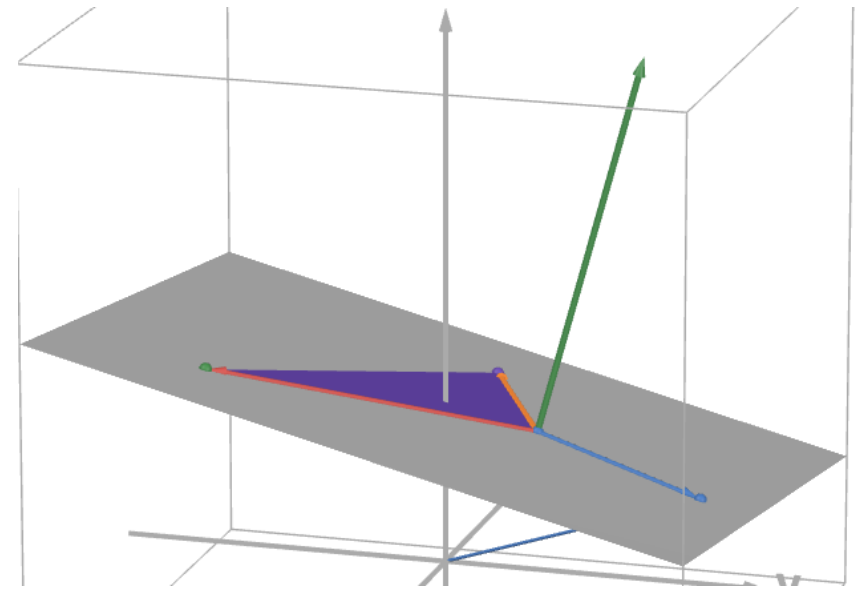
Let (x, y, z) be any point on the plane.

Let $\mathbf{r} = \langle x, y, z \rangle$ be the “position” vector that points from the origin to this point.

To describe a plane, we want...

A normal vector (perp. to the plane) and an initial position (pt on the plane).

- $\mathbf{n} = \langle a, b, c \rangle$
- $\mathbf{r}_0 = \langle x_0, y_0, z_0 \rangle$



Key idea:

The vector $\mathbf{r} - \mathbf{r}_0$ lies parallel to the plane, so it is perpendicular to $\mathbf{n}$.

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$$

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Plane Equations and Intercepts Part 2 HW #1–4, #8

Example: Find the plane through

$$P(2,0,0), Q(0,3,0), R(0,0,6)$$

Follow-up:

Can you visualize the plane using its intercepts?

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Relationships Between Planes Part 2 HW #5–6

General Plane Facts

1. Two planes are parallel if their normal vectors are parallel.
2. Non-parallel planes intersect in a line.

Lines and Planes Together Exam-style application

Example:

Where does the line through $(1,0,1)$ and $(4,-3,5)$ intersect the plane

$$x + y + z = 10$$

Follow-up:

Can you give the equation for the line in more than one way?

Does changing the line parameterization change the intersection point?